AGRISCIENCE AND TECHNOLOGY
Armenian National Agrarian University
ԱԳՐՈԳԻՏՈՒԹՅՈՒՆ ԵՎ ՏԵԽՆՈԼՈԳԻԱ АГРОНАУКА И ТЕХНОЛОГИЯ
International Scientific Journal
ISSN 2579-2822

Journal homepage: anau.am/scientific-journal
doi: 10.52276/25792822-2022.4-341

UDC 631.352.022

# Rheological Basics of the Adjusted Theory on the Plant Stems Sliding Cutting

**A.V. Altunyan, A.P. Tarverdyan**
*Armenian National Agrarian University*
artur_altunyan@mail.ru, arshaluystar@gmail.com

## ARTICLE INFO

***Keywords:***
*plant raw material,*
*mathematical model,*
*rheological properties,*
*slide cutting,*
*minimum energy consumption*

## ABSTRACT

The second article of the series considers the method of theoretical solution to the problem of plants stems sliding cutting with the account for the rheological properties of the material. In the result of solving the differential equation for the behavior of rheological model selected for the stem, the normal and tangential stresses at the contact zone of blade and stem interaction have been determined.

The basic stresses have been determined in the result of the stress states analyses. The obtained results enable to identify the optimal geometric and kinematic parameters, in case of which the sliding cutting can be implemented with minimum energy consumption.

## Introduction

Rheological properties of the raw materials and products are vital in the harvesting, raw material treatment, processing and selling procedures in the field of agricultural production, particularly in the plant growing and pomicultural sectors. Such an approach has taken root in recent times and is gradually being strengthened both through theoretical and experimental research.

The main prerequisite for increasing the efficiency of agricultural production is the complete mechanization of the above-mentioned processes; besides, the following requirements are usually presented to the machines implementing the technological processes: effective and high-quality implementation of technological processes with minimal energy consumption.

The mentioned issue is possible to solve only in case when the indices characterizing physico-mechanical, rheological and chemical properties of the developed and influenced environment are taken as a background while developing, designing and calculating such kind of machines. This approach was developed at the start of the 20th century (Goryachkin,1965) and was further improved by a number of researchers (Zheligovskiy, 1941, Reznik, 1975, Tarverdyan, 1996, Osobov and Noreiko, 1984, Rehkugler, 1966). In the current article an attempt is made to carry out force analysis for the plant stems cutting process taking into account the properties of the material being cut upon their rheological modeling.

Based on the approach that real materials exhibit relaxation and creep properties in the cutting process (Reznik, 1975, Tarverdyan, 1996, Osobov and Noreiko, 1984, Diamante and Umemoto, 2015), we'll try to introduce the blade and stem stress interaction procedure through some model known in rheology. The axiom well-known in rheology can be assumed as a justification, according to which "any real body is endowed with rheological properties expressed in different degrees" (Reiner, 1965). Multiple scientific research works have been accomplished upon the application of the above stated principle (Altunyan, 2009, Diamante and Umemoto, 2015, Kaliyan and Morey, 2009, Faborode and O'Callaghan, 1989, Dowgiallo, 2005).

The need for a more comprehensive study of the plants stem cutting has been justified in another research work (Tarverdyan and Altunyan, 2022) with the aim of identifying the design and calculation of cutting devices for more efficient, energy saving and reliable harvesters and mowers. Numerous research works have confirmed that the most energy-efficient method of cutting plant stems is the oblique sliding cutting (Goryachkin, 1965, Zheligovskiy, 1941, Tarverdyan, 1996). It is noteworthy that the mentioned statement is mainly based on the empirical data, whereas it is obvious that the nature of cutting phenomenon can be fundamentally revealed through theoretical research. An important prerequisite for the study of plants stems cutting is the determination of the main indices of the stems physicomechanical properties the latter coming forth as an impact environment. This circumstance has been mentioned by a number of field-specific researchers (Reznik, 1975, Tarverdyan, 1996, Rehkugler, 1966, Altunyan, 2009, Faborode and O'Callaghan, 1989, Dowgiallo, 2005).

Long-term studies on the plants stems anatomical and morphological structure, physicomechanical properties of the material, different ways and principles of their cutting process (Tarverdyan, 1996, Altunyan, 2009, Tarverdyan, 2004, Filin, 1975) give ground to insist that a more precise solution to the problem of cutting the stems as anisotropic composite body with the knife blade, should be sought in the accurate selection of their rheological models. Such an approach makes it possible to investigate the complete indicators of the physico-mechanical states of the body under study, such as stresses, deformations, changing speeds of stresses and deformations and other manifestations (Filin, 1975, Tarverdyan, 2004, Tarverdyan, 2014).

The aim of the current research work is to study the stress-strain state of the plant stems cutting process representing the properties of the developed environment via rheological modeling.

In this respect, the study of the problem concerning the sliding cutting via blade and enhancing the specified solution thereto is quite justified both from theoretical and practical perspectives.

## Materials and methods

In the result of long-term studies of anatomical and morphological structure of the spiked cereal stems and the physicomechanical properties of their tissues (Tarverdyan, 1996), it has been grounded that, with some approximation, an assumption can be made that the rheological model of the stem is a model of elasto-plastic adhesive dense medium, which is known as "Bingham body" (Figure 1).

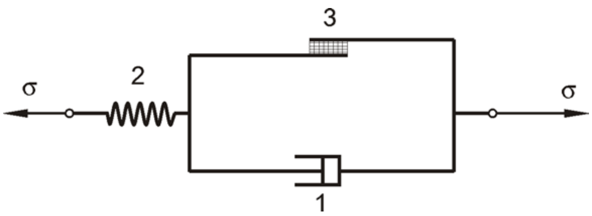


**Figure 1.** Diagram of the plant stem rheological (elastoplastic adhesive) model *(composed by the authors).*

It is a combination of Hooke's solid body (2), Newtonian fluid (1) and Saint-Venant or Prandtl body (3) as a mechanical model of rheological body with more complicated properties.

The rheological behavior of the selected body is described through the following equations [8]:

$$\sigma=\varepsilon\cdot E,\ \text{where } \sigma\leq\sigma_h$$
$$\sigma=\sigma_h+\eta\left(\frac{d\varepsilon}{dt}-\frac{1}{E}\cdot\frac{\partial\sigma}{\partial t}\right),\ \text{where } \sigma>\sigma_h, \quad (1)$$

where $\sigma$ is the actual total stress in the body being cut (stem), $\varepsilon$ is the relative deformation of the material, $E$ is the elasticity modulus of the material, $\sigma_h$ is the yield point of the material in case of one-dimensional deformation, $\eta$ is the coefficient of comparability, which depends on the deformation speed, in physical terms it is viscosity coefficient with $\left[\frac{N\cdot s}{m^2}\right]$ measurement.

In the case when the relative deformation increases uniformly $\frac{d\varepsilon}{dt}$ is the speed of the relative deformation (V). It is known that:

$$\varepsilon=\frac{\Delta\ell}{\ell},\quad \frac{d\varepsilon}{dt}=\frac{1}{\ell}\cdot\frac{d\Delta\ell}{dt}, \quad (2)$$

where ℓ is the thickness of the cut material (stem diameter), $\Delta\ell$ is the absolute deformation of the material or the blade's path crossed throughout the material.

If it is assumed, that the change of the absolute deformation value, which takes place during a very short time, is uniform, then in the (2) equation we can assign:

$$\frac{d(\Delta l)}{dt} = V,$$

hence

$$\frac{d\varepsilon}{dt} = \frac{V}{l}, \tag{3}$$

where $V$ is the absolute deformation speed of the material. On the other hand, $V$ is the equivalent speed, which occurs when implementing cross/direct cutting or cutting without sliding, therefore: $V=V_b$.

By inserting (3) into (1) and conducting some modifications, we get the following:

$$\sigma E + \eta\frac{\partial\sigma}{\partial t} = E\sigma_h + \eta E\frac{V}{l}\cdot \tag{4}$$

In the abovementioned equation $V$ is the absolute deformation speed of the material being cut, which in case of direct cutting, according to the direction, coincides with the vectors of stresses, blade cutting force and cutting speed (absolute cutting) coefficients, anyhow, in case of sliding cutting the vector of cutting speed coefficient deviates from the normal by the slip angle   (Figure 2).

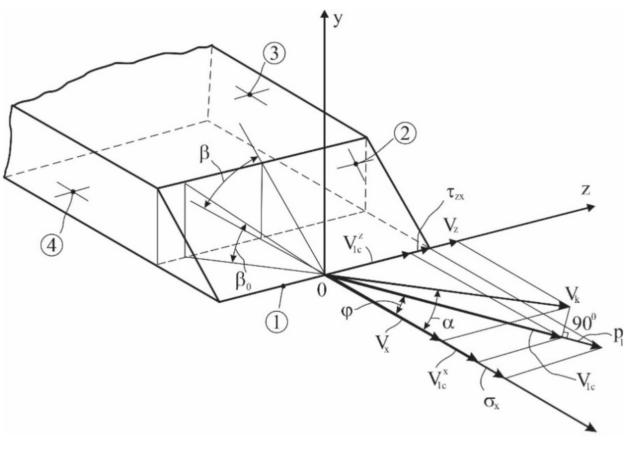


**Figure 2.** The diagram of stresses identification in the zone of knife blade (1) and stem interaction in case of sliding cutting *(composed by the authors).*

**Results and discussions**

The blade cutting resistance of the plant-based solid body has been determined per the (4) equation, considering the force interaction of the cut body with the cutting blade edge (I), facet (II) (by the example of unifacial blade) and the blade spine (III and IV) (Figure 2). Considering that in case of sliding cutting, there is slip angle  and that the directions of stress and velocity coefficients don't coincide, the relation of the absolute deformation speed vector with the cutting speed looks as follows:

$V=V_c=V_r\cdot cos(\tau-\varphi)$, ($\varphi$ is the friction angle).

The general form of the solution of the obtained first order linear differential equation (4) is as follows:

$$p = \sigma_h + \eta\frac{V}{\ell} + C\cdot e^{-\frac{Et}{\eta}}, \tag{5}$$

where the coefficient of stress arising in the cutting process is denoted by p instead of $\sigma$ given that in further judgements the normal stress component will be denoted by $\sigma$.

The expression (5) serves as a background to identify the stresses in the mentioned descriptive blade section (Figure 2).

For the blade edge (section l) the base data (initial condition) are as follows:

when $t=0$, $p_l = \sigma_h$, hence: $p_l = \sigma_h + \eta\frac{V}{\ell} + C$ wherefrom by inserting the value of $C$ of $\mathrm{C} = -\eta\frac{V}{l}$ in (5), we'll have the following:

$$p_l = \sigma_h + \eta\frac{V_1}{\ell_1}\left(1 - e^{-\frac{Et}{\eta}}\right), \tag{6}$$

where $\sigma_h$ is the yield point of the material, thus, it is constant during the cutting deformation, whereas the second summable of the expression is the dynamic stress component.

As mentioned in the above discussed problem it can be assumed that the direction of total stress vector *(p)* coincides with the vector of deformation or cutting speed *($V_c$)*, hence, it is also deviated from the vector of blade shift speed coefficient *($V_b$)* upon the angle of *($\alpha$-$\varphi$)* (Figure 2).

The normal stress value for the blade edge (I) will be:

$$\sigma_x = p_l\cdot\cos\varphi = \left[\sigma_h + \eta\frac{V_{cx}}{\ell_1}\left(1 - e^{-\frac{Et}{\eta}}\right)\right]\cdot\cos\varphi. \tag{7}$$

Frictional stress along the blade edge will be:

$$\tau_{zx} = p_l \cdot \sin\varphi = \left[\sigma_h + \eta\frac{V_{cz}}{\ell_1}\left(1 - e^{-\frac{Et}{\eta}}\right)\right]\cdot \sin\varphi. \qquad (8)$$

As mentioned above the total stress *(p)* in the discussed situation has the same direction with the cutting speed vector *($V_c$)*, which is deviated from the direction of blade shift or external impact force upon *(α-φ)* angle. It follows herefrom that the transitional link from the total stress to the external cutting force *(P)* looks as follows:

$$\mathrm{P} = \frac{p\cdot A}{\cos(\alpha-\varphi)} \quad \text{or}$$

$$P = A\cdot\left[\sigma_h + \eta\frac{V}{\ell}\left(1 - e^{-\frac{Et}{\eta}}\right)\right]\cdot\frac{1}{\cos(\alpha-\varphi)}, \qquad (9)$$

where $A$ is the contact surface of the blade edge and the material being cut.

To determine the interaction force factors emerged on the blade facet (II), let's select rectangular coordinate system *OXYZ*, *Z* and *X* axes of which are coincident with the facet plane (*XOZ* plane coincides with the facet plane), while *Y* axis is perpendicular to the plane facet (Figure 3).

In other words, the *OXYZ* system depicted in Figure 2 has rotated round the *Z* axis upon angle (in Figure 3 the mentioned system is *O*).

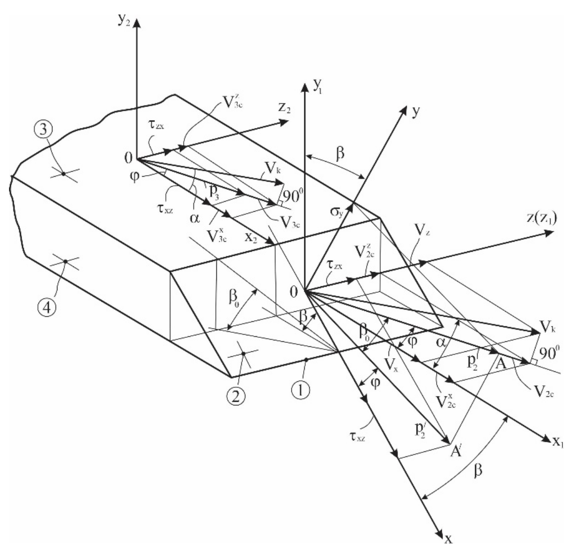


**Figure 3.** The diagram of contact stresses identification at the interaction plane of blade facet and blade spine with the stem in case of sliding cutting *(composed by the authors).*

The vectors of blade speed *($V_b$)*, cutting speed *($V_c$)* and total stress are situated in the same $X_1OZ$ horizontal plane (Figure 3).

If projecting $p_2$ vector of total stress towards the directions of *X, Y* and *Z*, we'll get:

$$\begin{cases} \sigma_y = \overline{AA^l} = p_2\sin\beta_0; \quad (\sigma_x = \sigma = 0), \\ \tau_{xy} = \overline{OA^l}\cos\varphi = p_2\cos\beta_0\cdot\cos\varphi, \\ \tau_{yx} = \overline{OA^l}\sin\varphi = p_2\cos\beta_0\cdot\sin\varphi, \end{cases} \qquad (10)$$

where $\beta_0$ is the facet gradient, transformation angle of $\beta$ angle, it is determined through the following expression: $\beta_0 = arctg(tg\beta\cdot\cos\alpha)$; $p_2$ is the total stress of the blade facet and the material being cut and it is determined through the same expressions as in case of $p_l$:

$$p_2 = \sigma_h + \eta\frac{V_2}{\ell_2} + C_2\cdot e^{-\frac{Et_2}{\eta}}. \qquad (11)$$

Upon the initial condition $C_2$ constant is determined:

if $t_2 = 0$ : $p_2 = p_l$, then:

$$\sigma_h + \frac{\eta V_1}{\ell_1}\left(1 - e^{-\frac{Et_1}{\eta}}\right) = \sigma_h + \eta\frac{V_2}{\ell_2} + C_2, \text{ where from,}$$

when placing $C_2 = \frac{\eta V_1}{\ell_1}\left(1 - e^{-\frac{Et_1}{\eta}}\right) - \frac{\eta V_2}{\ell_2}$ in (11), we'll get:

$$p_2 = \sigma_h + \frac{\eta V_2}{\ell_2} + \left[\frac{\eta V_1}{\ell_1}\left(1 - e^{-\frac{Et_1}{\eta}}\right) - \frac{\eta V_2}{\ell_2}\right]\cdot e^{-\frac{Et_2}{\eta}}. \qquad (12)$$

On the blade facet for the normal and frictional stress we'll have:

$$\begin{cases} \sigma_y = \left\{\sigma_h + \frac{\eta V_{2y}}{\ell_2} + \left[\frac{\eta V_{2y}}{\ell_1}\left(1 - e^{-\frac{Et_1}{\eta}}\right) - \frac{\eta V_{2y}}{\ell_2}\right]\cdot e^{-\frac{Et_2}{\eta}}\right\}\sin\beta_0, \\ \tau_{xz} = \left\{\sigma_h + \frac{\eta V_2}{\ell_2} + \left[\frac{\eta V_{2x}}{\ell_1}\left(1 - e^{-\frac{Et_1}{\eta}}\right) - \frac{\eta V_{2x}}{\ell_2}\right]\cdot e^{-\frac{Et_2}{\eta}}\right\}\cos\beta_0\cdot\cos\varphi, \\ \tau_{zx} = \left\{\sigma_h + \frac{\eta V_2}{\ell_2}\left[\frac{\eta V_{1z}}{\ell_1}\left(1 - e^{-\frac{Et_1}{\eta}}\right) - \frac{\eta V_{2z}}{\ell_2}\right]\cdot e^{-\frac{Et_2}{\eta}}\right\}\cos\beta_0\cdot\sin\varphi. \end{cases} \qquad (13)$$

In the upper and lower planes of the blade spine (III and IV zones) the mechanical picture of interaction with the body being cut is identical. In Figure 3 the diagram of speeds, total stresses and their components is designed only for the upper plane.

For the plane, the solution of differential equation looks as follows:

$$p_3 = \sigma_h + \eta \frac{V_3}{\ell_3} + C_3 \cdot e^{-\frac{Et_3}{\eta}}. \quad (14)$$

The constant of $C_3$ integration is determined from the following initial condition:

when $t_3=0$, $p_3=p_2$, wherefrom it follows:

$$C_3 = \frac{\eta V_2}{\ell_2} + \left[\frac{\eta V_1}{\ell_1}\left(1-e^{-\frac{Et_1}{\eta}}\right) - \frac{\eta V_2}{\ell_2}\right] \cdot e^{-\frac{Et_2}{\eta}} - \frac{\eta V_3}{\ell_3}.$$

When placing the value of $C_3$ in (14), we'll get:

$$p_3 = \sigma_h + \eta \frac{V_3}{\ell_3} + \left\{\frac{\eta V_2}{\ell_2} + \left[\frac{\eta V_1}{\ell_1}\left(1-e^{-\frac{Et_1}{\eta}}\right) - \frac{\eta V_2}{\ell_2}\right] \cdot e^{-\frac{Et_2}{\eta}} - \frac{\eta V_3}{\ell_3}\right\} \cdot e^{-\frac{Et_3}{\eta}}. \quad (15)$$

The frictional stresses in the blade spine planes are determined in the following way:

$$\tau_{xz} = p_3 \cdot \cos\varphi, \qquad \tau_{zx} = p_3 \cdot \sin\varphi. \quad (16)$$

When determining the cutting resistance forces via stresses in the blade planes (III and IV), the results obtained from (12) expressions should be doubled, so as to consider the impact of both planes.

When determining stresses caused throughout the cutting process by the (7), (8), (11) and (12) expressions, the speed components in the specific blade zone are marked, and since the blade speed *($V_b$)* is known in each specific problem, the components are also known:

$$V_{1x}^c = V_{3x}^c = V_c \cos\varphi, \quad V_{1z}^c = V_{3z}^c = V_c \sin\varphi,$$

$$V_{1c} = V_{3c} = V_b \cos(\alpha - \varphi), \quad V_{2y}^c = V_{2c} \sin\beta,$$

$$V_{2x}^c = V_{2c} \cos\beta \cdot \cos\varphi, \quad V_{2z}^c = V_{2c} \cos\beta \cdot \sin\varphi,$$

$$V_{2c} = V_b \cos(\alpha - \varphi) \cdot \cos\beta_0.$$

In the stress determination expressions the main variable is the cutting time, since the cutting depth is also a function dependent on time $\ell = f(t)$, hence when making practical caculations with the derived theoretical expressions, it is necessary to denote time with seconds in $0 \le t \le t_4$ interval, where $t_4$ is the time required for the complete stem cutting.

As it was already mentioned in the previous series of the article (Tarverdyan and Altunyan, 2022), where the cutting process was viewed as a forced crack propagation, as a result of theoretical research, expressions were derived which enable to identify the basic stresses in the already known zones of the blade. To compare the recommended two approaches, let's leave the base data unchanged. The material to be cut is the stem of "Bezostaya-1" wheat variety in its full maturity period, the stem diameter is $d=4\cdot10^{-3}$ m, the yield point (flow point) of the stem matter $\sigma_y=270$ MPa, elasticity modulus - E=$6\cdot10^4$ MPa, strength limit - $\sigma_h=320$ MPa, frictional coefficient - $tg\alpha=0.31$, the acute angle of the blade facet - $\beta=18^0$ blade speed - $V_b=20$ m/s, $\eta=4\cdot10^{-3}$ MP*s; $\alpha=68^0$; $0 \le t \le t_c$ $(t_c)$ is the stem cutting duration $t_c = 2\cdot10^{-4}$ s. By placing the base data in the (7), (8), (13), (15) and (16) expressions, the force factors of blade and cutting stem interaction are determined in the form of normal and frictional stresses in the mentioned descriptive blade zones.

In this option of problem solution, the following numerical values for the basic stresses caused in the material being cut at the descriptive blade zones/sections have been derived:

$\sigma_{1(I)}=387.0$ MPa, $\sigma_{1(II)}=288.0$ MPa,
$\sigma_{1(III)}=\sigma_{1(IV)}=256.0$ MPa.

In the previous options of the problem solution, the following numerical values were obtained for the same units:

$\sigma^*_{1(I)}=430.0$ MPa, $\sigma^*\sigma_{1(II)}=315$ MPa,
$\sigma^*_{1(I)}=\sigma^*\sigma_{1(II)}=275.0$ MPa.

Comparing the results of the two options it can be concluded, that though the results differ by about 10 %, their changing regularity is almost the same, so

$\sigma^*_{1(I)}:\sigma^*\sigma_{1(II)}=1.365$, $\sigma_{1(I)}:\sigma\sigma_{1(II)}=1.344$,

$\sigma^*_{1(II)}:\sigma^*_{1(III)}=1.145$, $\sigma_{1(III)}:\sigma\sigma_{1(III)}=1.125$,

$\sigma_{1(I)}:\sigma_{1(III)}=1.563$ $\sigma_{1(I)}:\sigma\sigma_{1(III)}=1.512$.

It follows wherefrom that both solution methods correctly address the mechanics of cutting process. Whereas, which of the methods is more precise will become clear in the result of experimental research.

## Conclusion

1. The normal and frictional stresses in the cutting zone of the material, on the planes of blade edge, facet and on both planes of the blade spine in case of sliding cutting have been determined through the integration of differential equation describing rheological behavior of the selected model for the stem of spiked cereal crops.

2. The normal and frictional stresses identified in the interaction zone of blade and cutting material have been viewed as force factors for the cutting material and in the

result of stress state analysis in the relevant zones/sectors, the basic stresses have been identified and the conditions under which the material is decomposed upon the effect of possibly minimum external factors have been set up.

3. Collating the two options of problem solution related to the stem sliding cutting (in the first option considering cutting as a forced crack propagation and in the second one – as a dynamic process in view of rheological properties of the material), it can be stated that though there is about 10 % incompatibility between the results of the mentioned methods, their changing patterns in the sectors of blade edge and spine planes are identical.

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*Accepted on 10.10.2022*
*Reviewed on 01.11.2022*