Theoretical Justification of the Dynamic Parameters in Plant Stems Sliding Cutting

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ABSTRACT

The first article of the series considers the opportunities of finding theoretical solutions to the issue of plants stem cutting via sliding method with account for the rheological properties of the substance.

Throughout the problem solution, cutting process was viewed as a transitional step for forced crack propagation in the body. Analytical expressions have been derived, which enable to analyze the stress state of the material in the cutting zone, to identify the principal stresses causing abrasions in case of sliding cutting. Intensity coefficient, sliding coefficient and their value domains have been identified, whereby the sliding cutting is implemented with minimum energy consumption.

Introduction

The theory developed through the scientific experimental data serves as a base for conducting research on the cutting process of the plants’ different organs, particularly plants stem cutting with blades. Some of the founders of the mentioned theory are the academician V.P. Goryachkin (Goryachkin, 1965) and his successor – academician V.A. Zheligovsky (Zheligovsky, 1941), and this theory was surely later developed by a number of other scientists (Reznik, 1975, Tarverdyan, 1996, O’dogherty and Cale, 1986, Dowgiallo, 2005, etc.).

It is apparent that in order to disclose the cutting nature and to identify the optimal values of cutting process and hence, those of the kinematic and geometric parameters for blade, it is necessary to investigate the physicomechanical properties of the material. Though this approach was developed still in the start of the previous century, yet, intensive investigations in this direction has been initiated since 1970 (Klaus Dobler, 1972, Reznik, 1975, Osobov and Noreiko, 1984, Tarverdyan, 1996). The mentioned research works have enabled to make adjustments to the current outlooks, develop cutting theories and consequently update the cutting devices.

The long-term theoretical and scientific experimental research activities implemented in this area (Tarverdyan, 1996) have provided opportunities to precisely determine...
the physicochemical properties of slender-stemmed and rigid thick-stemmed plants at different developmental stages. By combining the research results of anatomical and morphological structure of stems and study results of the physicochemical properties of the given substance, mechanical models have been developed which helped make considerable adjustments and corrections to the analytical expressions in the cutting theory. Nevertheless, it is worth mentioning that in the investigations conducted thereto, mainly some constant values of the indicators characterizing the physicomechanical properties of the cutting material are assumed as a base, while throughout deformation process they undergo changes; in other words, the rheological properties of the material are not considered. In some research papers (Reznik, 1975, Tarverdyan, 1996, Tarverdyan, 2004, Altunyan., 2008) this issue was addressed but the results of the problem solutions are not finalized and generalized enough to set the optimal cutting parameters and to develop cutting apparatus operating with minimum energy consumption.

Upon the current and classical analysis of the stress states, rheological modeling methods of the materials can be added to the description of the material cutting process, where the provisions of elasticity theory are mainly used, which are of no less importance, but are hardly applied in the theory of plant-based raw stuff cutting.

Taking into account the abovementioned and also based on the further comparison of the obtained results, we find it relevant to introduce one more approach of the theoretical study for plants stem cutting, which will further enable to interpret the cutting problem on the principle of rheological modeling and with minimum energy consumption more comprehensively.

The problem of cutting with minimum energy consumption is solved for the individual case, that is, for the case of oblique front cutting (Tarverdyan, 2014), whereas, it is known that effective blade cutting is ensured in case of sliding cutting. It is noteworthy that the majority of current cutting devices cut the plants stems and other plant materials just in conditions of slide provision. In this regard the study of the problem related to the blade sliding cutting and its specified solution is quite justified and relevant from both theoretical and practical perspectives.

The goal of the current work is to study the stems cutting via blade sliding by analyzing the stress-strain state of the procedure and to substantiate the values of dynamic parameters providing minimum critical cutting force.

**Materials and methods**

Based on the afore stated facts an attempt has been made to adjust the stems blade cutting theory by applying the approach of analyzing the stress states of the material to be cut. The problem has been formulated and the expressions resulted from its solution, have enabled to determine the minimum value of the cutting force and the value of cutting speed, which ensures accurate blade cutting with minimum energy consumption.

It is necessary to consider that during the sliding cutting of stem (Figure 1), the stress-strain state of the material in the zone of the blade edge, i.e., at the tip of cutting (crack) (I), on the facet (II) and in both planes (III and IV) of the body (blade spine) are different, and hence, the basic stresses (σ) and extreme friction stresses $\tau_{\text{max}}$ on those planes are determined through various expressions.

![Figure 1. The simplest diagram for introducing the nature of sliding cutting (composed by the authors).](image)

In the discussed issue the interpretation and proper modeling of the objective picture for the stress-strain state in the direct interaction zone of the blade edge and the cut material (stem) is of vital significance.

The results of empirical research conducted through the high-speed/fast microscopic imaging of the stem cutting process (10 000 frames per second/fps) and synchronous recording of cutting force data (Tarverdyan, 1996) give grounds for stating that cutting process can be viewed as a forced crack (fracture) propagation with a certain velocity. When assuming such model, the problem of stem cutting with minimum energy consumption leads to the determination of the kinematic and dynamic optimal parameters upon the favorable conditions of crack (fracture) forced propogation in the body. It is worth mentioning that the introduced approach is appropriate to apply starting from the wax maturation phase of the plants stem development, since the rheological models of the stem matter are extremely different related to the developmental stages: elastic-plastic adhesive, elastic-plastic and elastic.
In the mechanics of solid body abrasion three cases of crack surface deformation are accepted (Figure 2).

It is vivid that in the mentioned three cases the shift of the crack planes towards each other is related to the stress state in the vicinity of the crack/cutting tip (Figure 3).

1. In case of regular tearing: 
\[ \begin{align*}
\sigma_x &= \frac{K_1}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[ 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right] \\
\sigma_y &= \frac{K_1}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[ 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right] \\
\tau_{xy} &= \frac{K_1}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} 
\end{align*} \]

(1)

\[ \sigma_z = 0 \] in case of flat stress state and \( \sigma_z = \nu(\sigma_x + \sigma_y) \) in case of flat deformation.

2. In case of latitudinal sliding: 
\[ \begin{align*}
\sigma_x &= -\frac{K_1}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \left[ 2 + \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \right] \\
\sigma_y &= \frac{K_1}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \left[ \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \right] \\
\tau_{xy} &= \frac{K_1}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[ 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right] 
\end{align*} \]

(2)

\[ \sigma_z = \nu(\sigma_x + \sigma_y), \quad \tau_{xz} = \tau_{yz} = 0. \]

3. Longitudinal sliding \((K_1 \neq 0; K_2 = K_3 = 0)\),
\[ \begin{align*}
\tau_{zy} &= \frac{K_1}{\sqrt{2\pi r}} \sin \frac{\theta}{2}, \\
\tau_{xz} &= -\frac{K_3}{\sqrt{2\pi r}} \cos \frac{\theta}{2}, \\
\sigma_x &= \sigma_y = \sigma_z = 0, \quad \tau_{xy} = 0. 
\end{align*} \]

(3)

In the (1), (2) and (3) expressions \( K_1, K_2 \) and \( K_3 \) are intensity coefficients for the above mentioned deformations, respectively and have specific role in force or energy estimation, \( \theta \) is the angle formed by the radius vector of the considered point with XOZ plane, \( r \) is the radius of the assumptive plastic zone in the front part of the cutting/crack. In the problem solution of stem blade cutting, the geometric linearization of the boundary conditions and linear theory of elasticity entail to stresses and infinite values/quantities of their gradients; then the concentration coefficient becomes meaningless.

The only way of problem solution is the disclosure of the strain state nature and intensity in the cutting zone, where the mechanism of material abrasion is concentrated. In this case, the only characteristics of stress distribution is the stress intensity coefficient – \( K \), which is not dependent on the coordinates of the points arranged in the cutting zone. Unlike concentration coefficient, the stress intensity coefficient is endowed with measurability \( \sqrt{N \cdot m} \).
Results and discussions

Based on the aforementioned interpretations and the circumstance that the intensity coefficient for each individual case is determined differently, the problem related to its determination is still unsolved and actual. A great number of problems for the determination of intensity coefficient with analytic methods are tackled, the detailed description and analysis of which are presented in the voluminous research work issued under the general editorship of G. Liebowitz (Liebowitz, 1975). Anyhow, it should be mentioned that those problems are solved mainly for infinite domains, while in conditions of finite values they produce insufficient results. The details about the described statements are introduced in the research work of A.V. Malik (Malik and Lavit, 2018).

Related to the issue under discussion, when it refers to the cutting extension in the plant material (stem), determination of intensity coefficient becomes a specific problem. This problem was previously addressed as well (Tarverdyan, 1996) and it was stated that among the expressions developed for the determination of intensity coefficient when solving stem cutting issue, the expression developed with the method recommended by D. Broek is the most preferable one, which considers the body finite sizes (Broek, 1974):

\[ K = Y \sigma \sqrt{\pi l}, \]

where \( Y \) is the polynomial considering the finite sizes of the cut body/material, \( \sigma \) is the regular stress in the front zone of cutting and \( l \) is the cutting depth.

The problem solving accuracy is mainly related to the accurate selection of \( Y \) polynomial and correct calculation of the coefficients. In our problem, the numerical coefficients of the polynomial have been determined empirically with the support of analytic expressions developed by Srawley-Gross (Srawley, et al., 1964) and Walsh’s methodology for finite cracks study (Walsh, 1971).

In the result of investigations polynomial’s expressions have been derived, which enable to determine the intensity coefficient of the knife blade cutting for both thin-stemmed and rigid thick-stemmed plants. Besides, these expressions provide opportunity to consider not only the changes in the cutting line length (stems’ latitudinal cross-section is a circle), but also, which is most important, to establish relationship between the cutting speed and intensity coefficient. By means of tensile deformation of the samples produced from the stems with preliminarily simulated cuttings at different depths, the numerical values of polynomial’s \( (Y) \) coefficients were determined (Tarverdyan, 2004). Particularly, the expression of intensity coefficient for the wheat stem looks as follows:

\[ K_1 = \sigma \sqrt{\pi l} \left[ 1.77 - 0.36 \frac{V_c t}{d} + 16.70 \left( \frac{V_c t}{d} \right)^2 - 34.34 \left( \frac{V_c t}{d} \right)^3 + 48.08 \left( \frac{V_c t}{d} \right)^4 \right], \]

where \( V_c \) is the cutting speed, \( t \) is the current cutting time \((0 \leq t \leq t_f)\), \( d \) is the diameter of the stem latitudinal cross-section.

From the \( K_1 \) expressions a significant conclusion is inferred, that is, the intensity coefficient is a variable value depending on time and cutting depth.

In case of sliding cutting the mentioned three modes of deformations take place simultaneously, and although under such circumstances the identification of the principal stresses and extreme friction stresses becomes complicated, after some allowable assumptions, the problem solution turns out to be simple. Taking into account that in case of sliding cutting \( \sigma \) and \( \tau_y \) have anyhow decisive roles and their maximum values are simultaneously fixed in case of \( \theta = 0 \) and also the fact that in case of abrasive deformation the radius of plastic zone is determined through the following expression (Broek, 1974):

\[ r = \frac{\sigma^3}{2 \pi \sigma_{\tau}}, \]

after some modifications, from the expressions of (1), (2) and (3), we’ll get:

\[ \sigma_y = \sigma_y = Y \sqrt{\sigma_{\tau}^2 + 0.5 \sigma^2}; \]

\[ \tau_{y\tau} = \frac{\tau_y}{\sigma} \sqrt{\sigma_{\tau}^2 + 0.5 \sigma^2}; \]

\[ \tau_{y\tau} = \frac{\tau_y}{\sigma} \sqrt{\sigma_{\tau}^2 + 0.5 \sigma^2}; \]

where \( Y = \frac{K}{\sigma \sqrt{\pi l}} \), \( \sigma, \tau_y, \tau_z \) are the external force factors in the interaction zone of knife blade and fracture of the cutting material (\( \sigma \) is the stress towards y axis, \( \tau_y \) and \( \tau_z \) are the friction stresses towards the directions of x and y respectively), \( \sigma_{\tau} \) is the flow point/limit of the material.

In the result of determining the basic stresses \( \sigma^3 + a \sigma^2 + b \sigma + C = 0 \) or solution of \( R^2 + pR + q = 0 \) equation through trigonometric method, the following expression has been derived for the principal stresses:

\[ \sigma_i = \frac{\sigma_y + \sigma_i}{3} \pm 2 R \cos \left[ \frac{\pi}{3} (i - 1) \right]. \]
where \( i=1,2,3, \) \( R=(Signa)\sqrt{\frac{P}{3} \cos 3\psi} = \frac{q}{2R^2}, \)
\[
P = \frac{Y}{3}(\sigma_{m}^+ + 0.5\sigma^2) \left[ 1 + \frac{3}{\sigma}(\tau_i^+ + \tau_i^-) \right],
\]
\[
q = \frac{Y}{3}(\sigma_{m}^- + 0.5\sigma^2) \left[ \frac{2}{9} \tau_i^+ + \frac{\tau_i^-}{\sigma^2} \right],
\]
hence:\[
R = \frac{Y}{3}(\sigma_{m}^+ + 0.5\sigma^2) \left[ 1 + \frac{3}{\sigma}(\tau_i^+ + \tau_i^-) \right],
\]
\[
\cos 3\psi = \frac{1 + \frac{4.5(\tau_i^+ + \tau_i^-)}{\sigma^2}}{\sqrt{1 + \frac{3(\tau_i^+ + \tau_i^-)}{\sigma^2}}}, \quad (7)
\]
where \( \psi \) is the angle formed by the main planes of the stress state with XOZ plane.

In the blade facet, the stress state of the material is also dimensional. The peculiarity of the mentioned zone consists in that the normal (\( \sigma \)) and friction (\( \tau \)) force factors are related via the following expression:
\[
\tau_i = \frac{t g \varphi_f}{\sigma} \cos \beta,
\]
where \( \varphi_f \) is the angle of the cut material and the blade surface friction, \( \beta \) is the blade acute angle.

In this case the expression of \( \cos \psi \) determination will have the following interpretation:
\[
\cos 3\psi = \frac{1 + \frac{4.5t g \varphi_f(1 + \varepsilon_i^2)}{\sigma^2}}{\sqrt{1 + \frac{3t g \varphi_f(1 + \varepsilon_i^2)}{\cos \beta}}}, \quad (9)
\]
In both platforms of the blade body (spine) the stress states are identical, while the relation of the friction stress (\( \tau_i \)) and regular stress (\( \sigma \)) is manifested through the following expression:
\[
\tau_i = \sigma \cdot t g \varphi_f. \quad (10)
\]
The expression for determination is as follows:
\[
\cos 3\psi = \frac{1 + \frac{4.5t g \varphi_f (1 + \varepsilon_i^2)}{\sigma^2}}{\sqrt{1 + \frac{3t g \varphi_f (1 + \varepsilon_i^2)}{\cos \beta}}}, \quad (11)
\]
By inserting the values of obtained units in (6) we’ll get the expressions of the principal stresses in the zones of blade edge, facet and body/spine.

\[
\sigma_{(i)} = m \left[ 1 + \sqrt{1 + \frac{3t g^2 \varphi_f(1 + \varepsilon_i^2)}{\sigma^2}} \cdot n \right], \quad (12)
\]
\[
\sigma_{(ii)} = m \left[ 1 + \sqrt{1 + \frac{3t g^2 \varphi_f(1 + \varepsilon_i^2)}{\cos \beta}} \cdot n \right], \quad (13)
\]
\[
\sigma_{(iii)} = \sigma_{(ii)} = m \left[ 1 + \sqrt{1 + 3t g^2 \varphi_f(1 + \varepsilon_i^2)} \cdot n \right], \quad (14)
\]
where \( m = \frac{2Y}{3} \sqrt{\sigma_{m}^+ + 0.5\sigma^2}, \) \( n = \cos \left( \psi + \frac{2\pi(i-1)}{3} \right) \)

From (12), (13) and (14) expressions of the principal stresses it can be inferred, that the changes of their values are related to the sliding coefficient (\( \varepsilon_i \)) and intensity coefficient (\( K \)), \( Y=f(t) \) through the polynomial’s correction function.

It should be taken into account that if there is an objective to determine the stress loads during the cutting process, then only the current (instantaneous) values of the stresses can be considered, since they are functions from the cutting depths (\( V_c, \tau \)). That parameter constitutes the expression of correction function \( Y=f(t) \), and hence, that of stress intensity coefficient. That is, when determining the stresses during the cutting process, the specified time moment should be definitely mentioned (\( t=t_i \)).

We find it relevant to introduce a number of numerical values for principal stresses estimated through this method to compare them with the results obtained via other theoretical and empirical ways.

The computations have been conducted based on the following baseline data: The stem of wheat variety Bezostaya-1 served as the material to be cut, which was at ripening stage with the diameter \( d=4 \times 10^{-3} m \), cutting speed was \( V_c=20 \ ms \), the friction coefficient of the stem substance and blade surface was \( t g \varphi_f = 0.31 \), the acute angle of blade facet \( \beta = 18^\circ \), the tensile strength of stem substance \( \sigma_{te}=320 \ MPa \), flow limit/fluidity \( \sigma_{te}=270 \ MPa \), \( \tau_c=180 \ MPa \).

In case of the abovementioned parameters the stem cutting time is \( t_i=2 \times 10^{-4} \) second, in the considered example let’s assume \( t=1 \times 10^{-4} \) s (in the middle of the cutting process). The most important factor of sliding cutting is \( \varepsilon_i=8 \) (the experiments have shown that the sliding cutting is most efficient in the domain of \( 6 \leq \varepsilon_i \leq 10 \)) (Tarverdyan, 1996, Tarverdyan, 2004).

In case of the mentioned numerical values the critical values of the stresses intensity coefficients make...
In the result of stress states analysis conducted for the discussed cutting zones, theoretical expressions have been derived, which enable to identify the principal stresses in the blade cutting edge, facet and body platform zones; besides, a relationship has been established between those stresses, intensity coefficient and the main descriptor of the sliding cutting, i.e., sliding coefficient.

It has been proved that the most significant index of the forced propagation of the crack/cutting \((K)\) – the stress intensity coefficient – which determines the values of the basic stresses, is reduced along with cutting depth increase \(\frac{V_c \cdot t}{d}\) and its maximum value complies with the start of sticking process of blade cutting edge into the body being cut. This is why the basic stress gets its maximum value at the start of cutting process, which considerably exceeds the limit strength of the cut body. Parallel with the cutting depth increase the basic stresses gradually decline.

In case of sliding cutting, the basic stresses are greatly affected by sliding coefficient \((\varepsilon_c)\) and together with its growth the principal stress increases as well. Theoretically it has been stated that in the domain of \(4.0 \leq \varepsilon_c \leq 14.0\) the growth of the basic stresses is intensive. The results of experiments have proved, that the optimal values of sliding coefficient are in the domain of \(6.0 \leq \varepsilon_c \leq 10.0\); in such conditions the required cutting force is 2.0 times lower than in case of regular frontal cutting. If \(\varepsilon_c > 10\), then, though the cutting force decreases, the cutting becomes inefficient due to the restriction of blade motion towards the regular direction.

References


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